Modeling Multi-state Diffusion Process in Complex Networks: Theory and Applications

Yishi Lin, John C.S. Lui (The Chinese University of Hong Kong)
Kyomin Jung (Seoul National University)
Sungsu Lim (KAIST)

Background: Diffusion in Networks

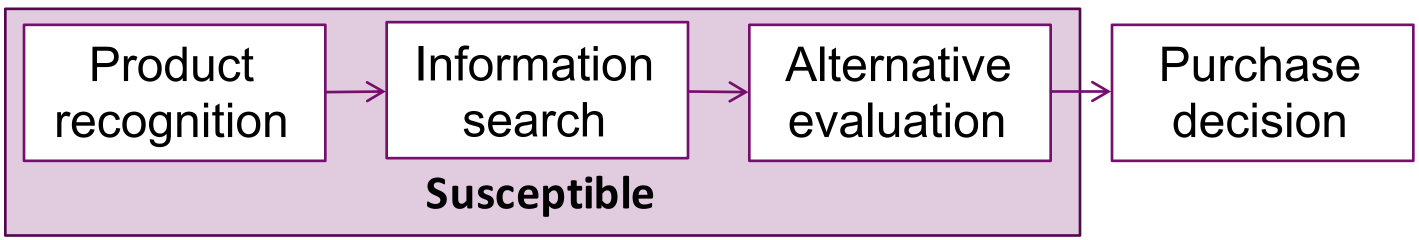
Diffusion in networks
- spread of idea, rumor, behavior, innovation, etc.

Predicting / Controlling the spread of information
- preventing the diffusion
  - e.g. rumor dissemination
- expediting the diffusion
  - e.g. viral marketing

Background: Motivation

Motivation I

- The traditional models usually assumed that the number of states is fixed and small. (too restrictive)
  - E.g. Consumer purchase decision process theory:



We propose a generalized SIS model that allows multiple susceptible states.

Background: Motivation

Motivation II

- In some situations, we need to consider competing sources.
  - e.g. competing products…

We model the behavior and dynamics of competing sources by extending our generalized SIS model.

Outline

Generalized SIS Model

- The ternary model as an example
- Numerical results

General SIS Model with Competing Sources

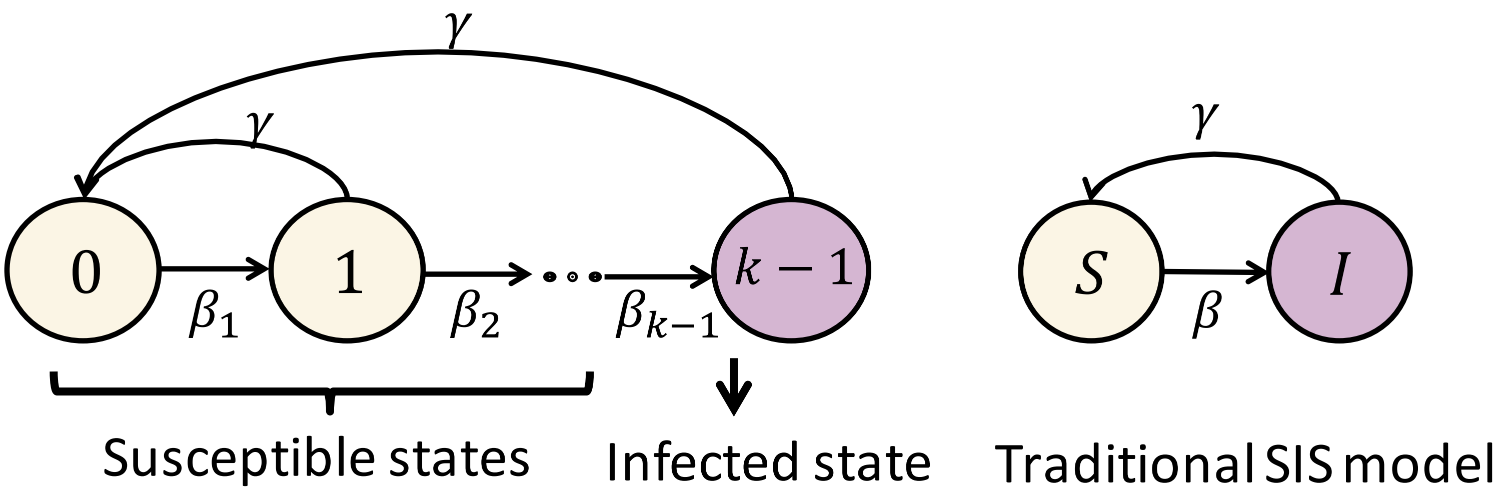
- The ternary model as an example
- Numerical results

Conclusion

General Multi-State SIS Model

The general k-state model

- Infection rate: $\beta_1, \beta_2, \ldots, \beta_{k-1}$
  - Each node in state $i < k - 1$ increases its state by 1 with a rate β_{i-1} if there is a contact with a node in state $k - 1$.
- Recovery rate: γ
  - Each non-zero state node turns to zero state with a rate γ.



Ternary Model ($k = 3$)

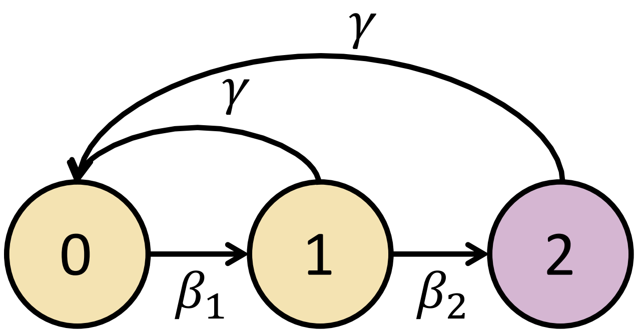
State 2 (infected) & State $1, 0$ (susceptible)

- $x_i(t)$: the fraction of nodes in state i
- Condition for equilibrium (x_0, x_1, x_2):

$$\frac{dx_2}{dt} = \beta_2 x_2 x_1 - \gamma x_2 = 0$$

$$\frac{dx_1}{dt} = -\beta_2 x_2 x_1 + \beta_1 x_2 x_0 - \gamma x_1 = 0$$

$$\frac{dx_0}{dt} = -\beta_1 x_2 x_0 + \gamma(1 - x_0) = 0$$



Ternary Model ($k = 3$)

Solving the above system of equations, we obtain

Case (i): **Trivial** equilibrium

- # infected nodes goes to 0 ($x_2 \to 0$)
- We show that it is a fixed point.

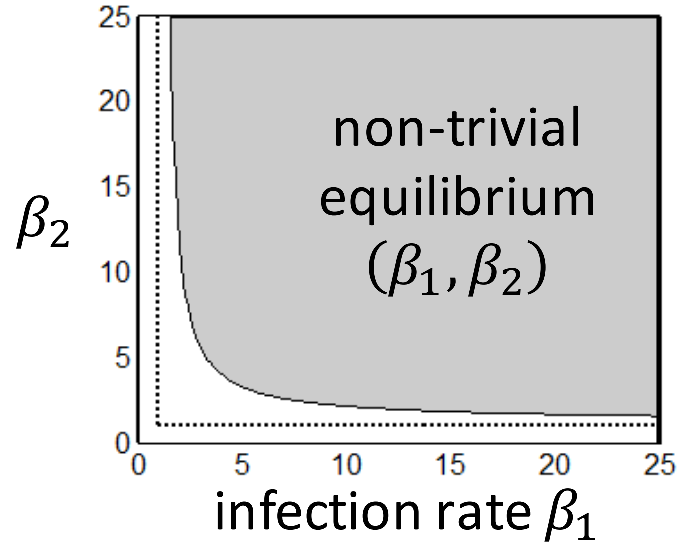
Case (ii): **Non-trivial** equilibrium

- $\left(x_2 \neq 0, x_1 = \frac{\gamma}{\beta_2}, x_0 = \frac{\gamma}{\gamma+\beta_1 x_2}\right)$

Ternary Model: Non-trivial Equilibrium

- For **Non-trivial** equilibrium, we show
  - The **necessary** conditions for the existence of non-trivial equilibrium: The "power" of infection (β_1, β_2) must be larger enough than the "speed" of recovery (γ).
  - The condition for stability of non-trivial solutions.

- We also extend this to k-state case and derive close form threshold for homogeneous infection rates (i.e. $\beta_1 = \beta_2 = \cdots = \beta_{k-1}$)



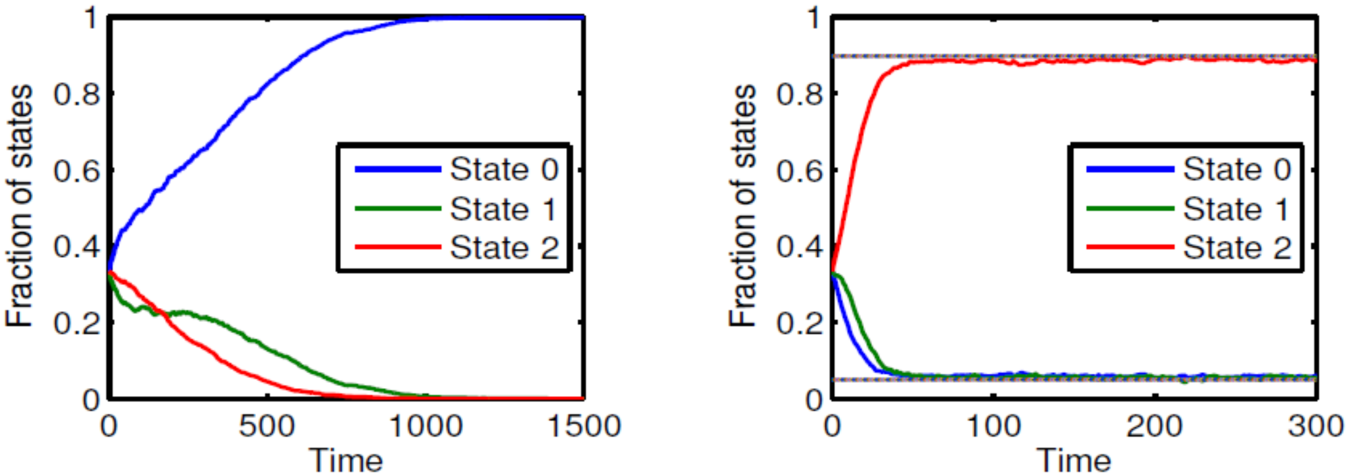
Numerical Experiments

Ternary model ($k = 3$)

- \# nodes $N = 5{,}000$
- Network 1: complete graph K_N
- Network 2: random ER graph $G(N, M)$
- Network 3: random power law graph $P(N, \theta, d, m)$

We show that our multidimensional mean-field results (the condition for the existence and stability of non-trivial solutions) hold for the above networks.

Numerical Experiments



(a) K_{5000}, $\beta_1 = \beta_2 = 3$ (b) K_{5000}, $\beta_1 = \beta_2 = 20$

The dotted line:
$(x_0, x_1) = (0.05, 0.897)$

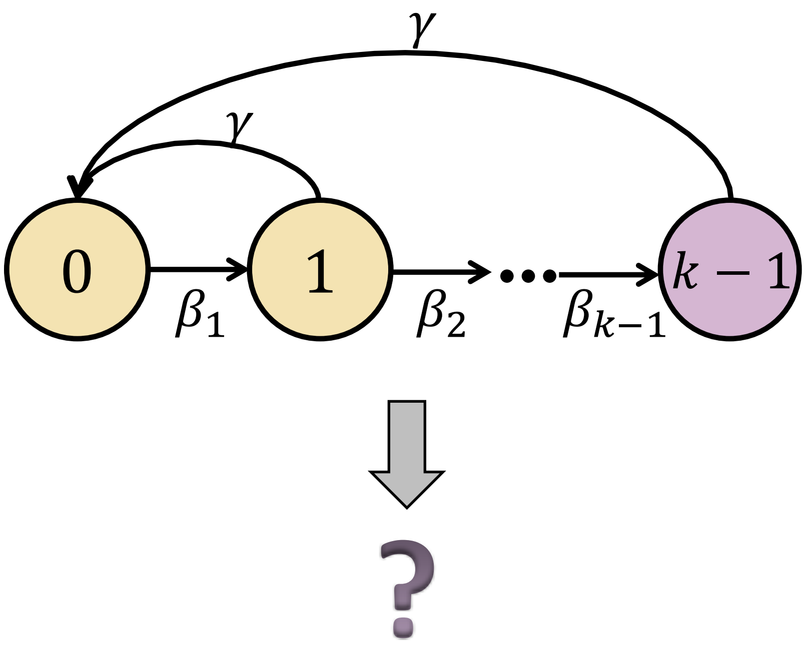
(stable Equilibrium)

Figure 3. Numerical results: The dynamics of x_0, x_1 and x_2 over time where $\gamma = 1$. (Initial fraction of nodes in different states: $x_0(0) = x_1(0) = x_2(0) = 1/3$)

General SIS model with competing sources

What will happen if there are competing sources?

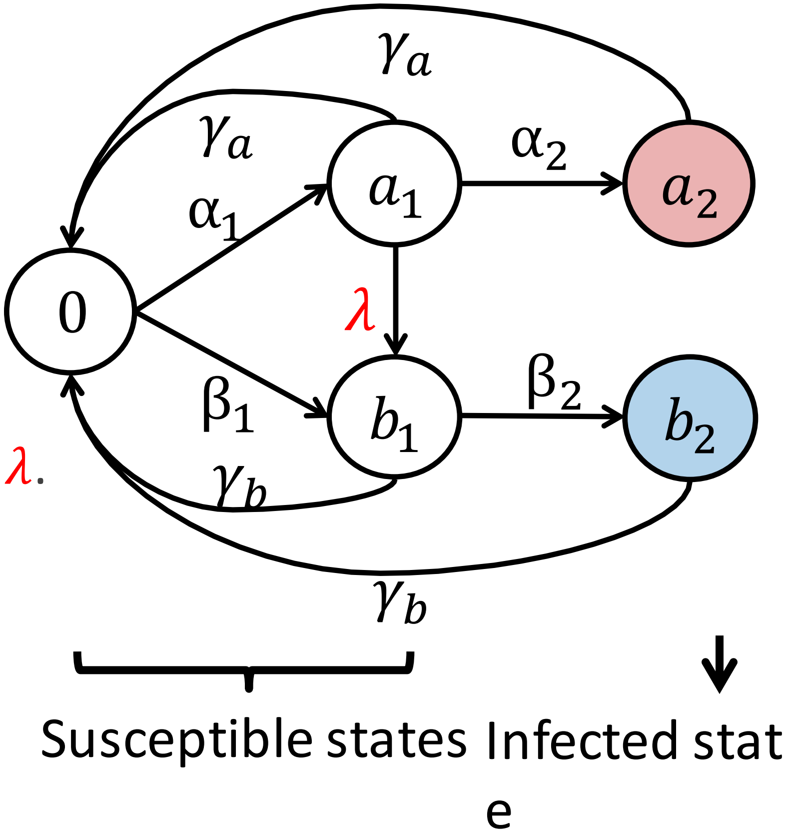
Suppose there exists two competing products, what will happen at the end?



Ternary model with competing sources

Ternary model

- Competing sources: a and b
- States set: $S = \{0, a_1, a_2, b_1, b_2\}$
- Infection rates: $\alpha_1, \alpha_2, \beta_1, \beta_2$
- Recovery rates: γ_a, γ_b
- Source b has **persuasive** power:
  - Nodes in state b_2 can change their neighboring nodes in a_1 to b_1 with rate λ.

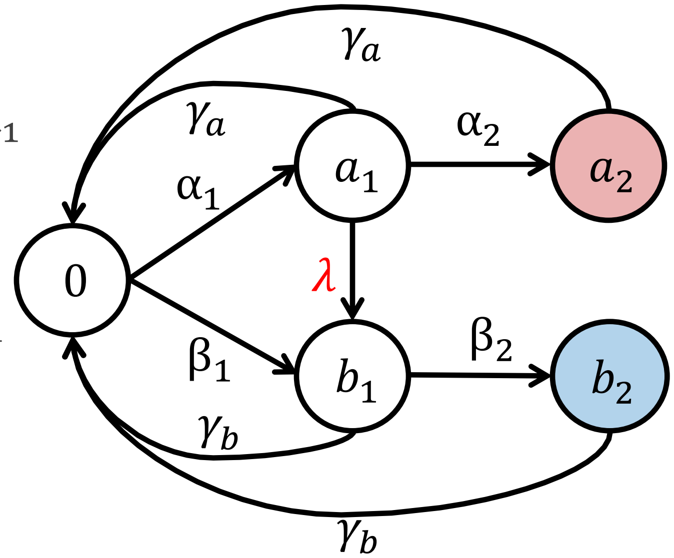


Ternary model with competing sources

Fraction of nodes in state s: $x_s(t)$

Dynamics of ternary model:

- $\frac{dx_{a1}}{dt} = \alpha_1 x_0 x_{a_2} - \alpha_2 x_{a_1} x_{a_2} - \lambda x_{a_1} x_{b_2} - \gamma_a x_{a_1}$
- $\frac{dx_{a_2}}{dt} = \alpha_2 x_{a_1} x_{a_2} - \gamma_a x_{a_2}$
- $\frac{dx_{b1}}{dt} = \beta_1 x_0 x_{b_2} - \beta_2 x_{b_1} x_{b_2} + \lambda x_{a_1} x_{b_2} - \gamma_b x_{b_1}$
- $\frac{dx_{b_2}}{dt} = \beta_2 x_{b_1} x_{b_2} - \gamma_b x_{b_2}$
- $x_0 = 1 - x_{a_1} - x_{a_2} - x_{b_1} - x_{\downarrow} b_2$



We solve it numerically and explore the dynamics of model with two competing sources.

Numerical Results

Compare the numerical results with the simulation results for ternary model

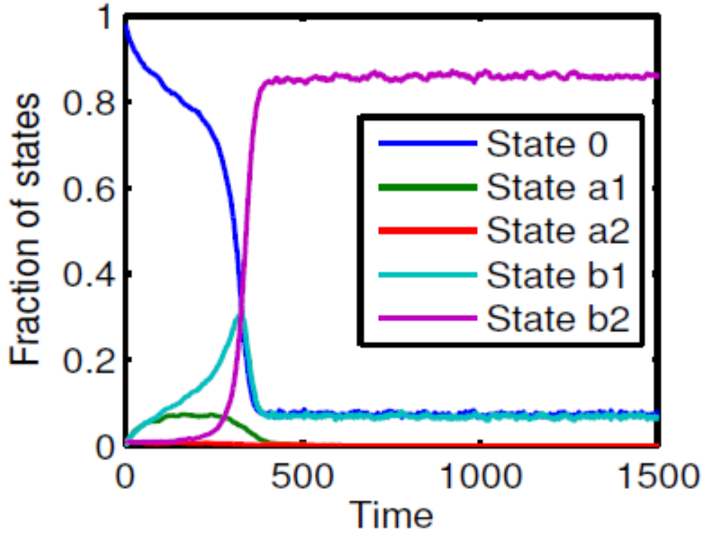
- Network 1: complete graph K_{5000}
- Network 2: random ER graph (1000 nodes)
- Network 3: random power law graph (1000 nodes)

Explore

- The impact of delay in deploying source b
- The impact of λ
- The impact of $\mathcal{M}$ and $\mathcal{N}$

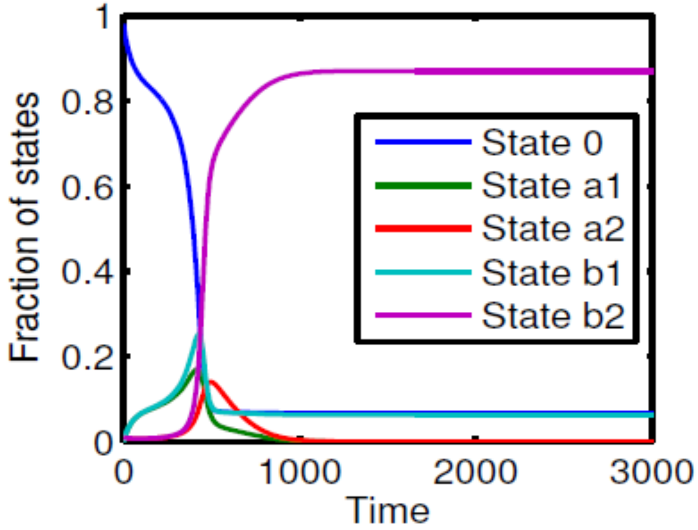
Simulation Results v.s. Numerical Results

Simulation results



K_{5000}, $\alpha_1=\alpha_2=\beta_1=\beta_2=16$, $\lambda=2$

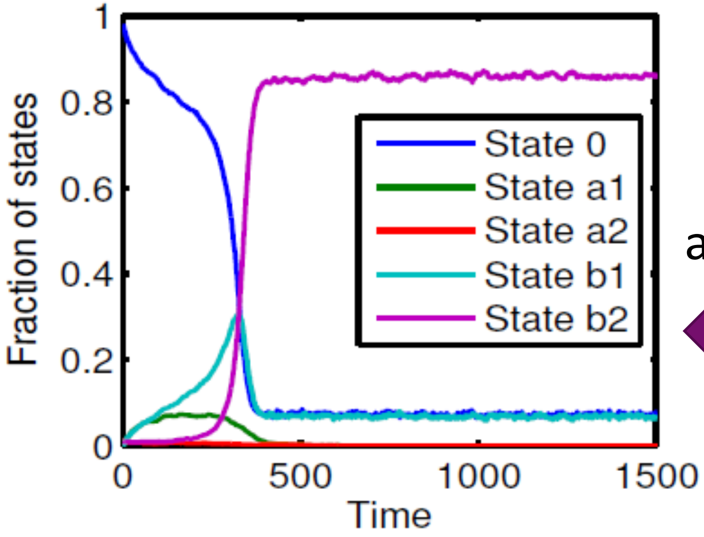
Numerical results



K_{5000}, $\alpha_1=\alpha_2=\beta_1=\beta_2=16$, $\lambda=2$

Simulation Results v.s. Numerical Results

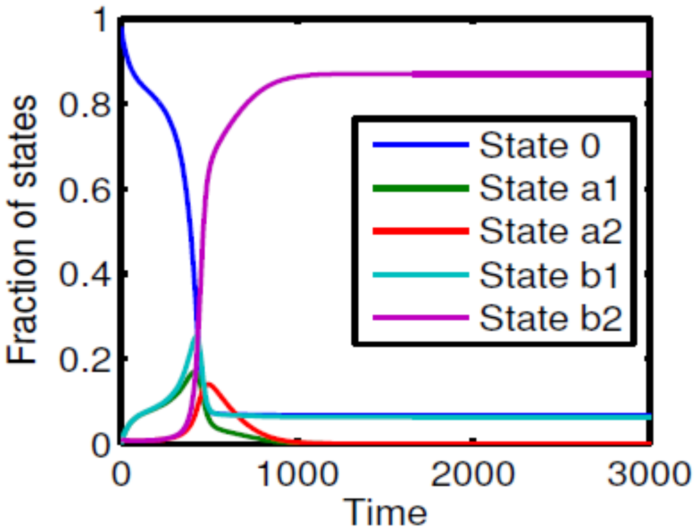
Simulation results



K_{5000}, $\alpha_1=\alpha_2=\beta_1=\beta_2=16$, $\lambda=2$

well approximate

Numerical results



K_{5000}, $\alpha_1=\alpha_2=\beta_1=\beta_2=16$, $\lambda=2$

The Impact of Delay in Deploying b

Suppose source a and b are two products.

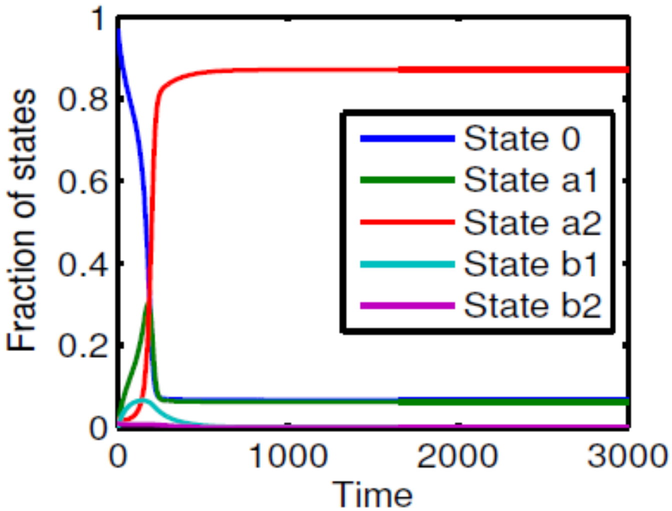
Product b is introduced to the market later than product a. **Will product b win?**

- $x_{b_2}(0) = \frac{1}{2} x_{a_2}(0)$

The Impact of Delay in Deploying b

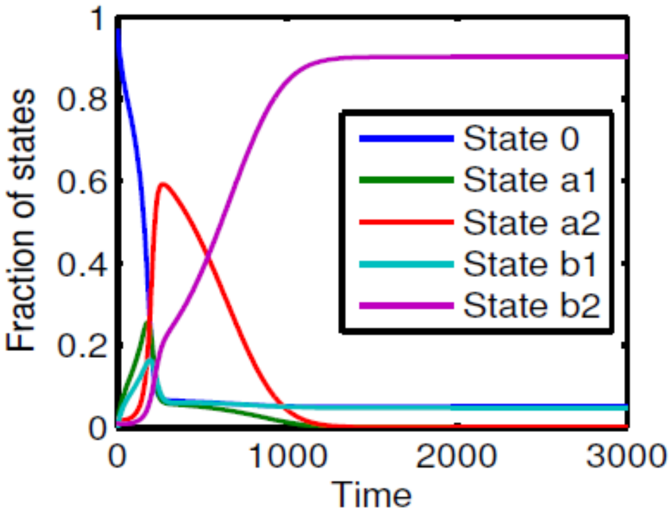
Product b is introduced later

- If it does have large enough infection power, it will lose.
- Otherwise, it may win.



(a) $\alpha_1=\alpha_2=\beta_1=\beta_2=16$, $\lambda=2$, $\gamma_b=1$

Not powerful enough!

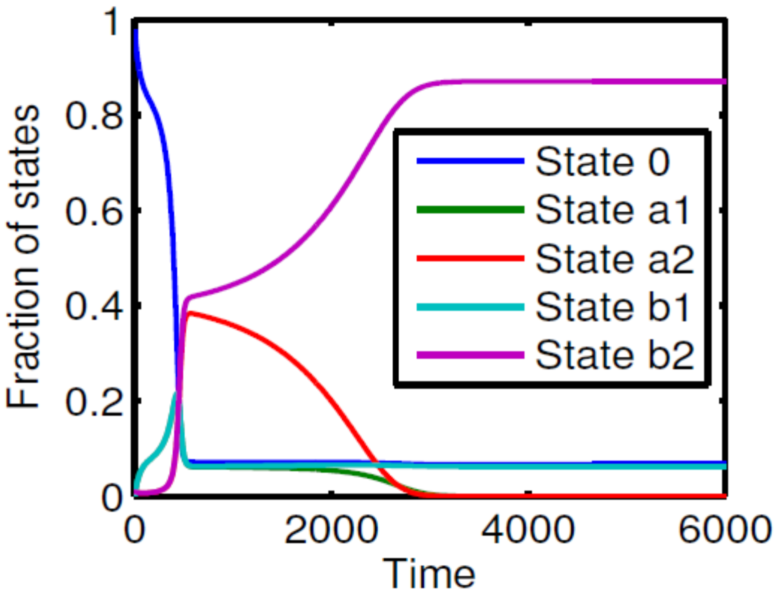


(b) $\alpha_1=\alpha_2=16$, $\beta_1=\beta_2=21$, $\lambda=2$, $\gamma_b=1$

Large infection rate!

The Impact of λ

Parameter λ captures the power of product a to attract potential buyers of product b.



(a) $\lambda = 0.1$,

Source a and b have the same infection and recovery rates.

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 16,$$
$$\gamma_a = \gamma_b = 1$$

Conclusion

In this work

- We propose a generalized multi-state SIS model that allows multi-intermediate susceptible states.
- We model the behavior of two competing sources, one dominant and one regressive, under the generalized SIS model.
- We show how different parameters of the model affect the information diffusion process and talk about the application of proposed models.

We believe that our work is a step towards elucidating the complex interactions between nodes in the epidemic spreading.

Thank you!

Extended Journal article:
Yishi Lin, John C.S. Lui, Kyomin Jung, Sungsu Lim. *Modeling Multi-state Diffusion Proce ss in Complex Networks: Theory and Applications*, Journal of Complex Networks, 2(4), 431-459, 2014.